

201-NYA-05 - Calculus 1
WORKSHEET: INTEGRALS

Evaluate the following indefinite integrals:

1. $\int (4x + 3) dx$
2. $\int (4x^2 - 8x + 1) dx$
3. $\int (9t^2 - 4t + 3) dt$
4. $\int (2t^3 - t^2 + 3t - 7) dt$
5. $\int \left(\frac{1}{z^3} - \frac{3}{z^2} \right) dz$
6. $\int \left(\frac{4}{z^7} - \frac{7}{z^4} + z \right) dz$
7. $\int \left(3\sqrt{u} + \frac{1}{\sqrt{u}} \right) du$
8. $\int \left(\sqrt{u^3} - \frac{1}{2}u^{-2} + 5 \right) du$
9. $\int (2v^{5/4} + 6v^{1/4} + 3v^{-4}) dv$
10. $\int (3v^5 - v^{5/3}) dv$
11. $\int (3x - 1)^2 dx$
12. $\int \left(x - \frac{1}{x} \right)^2 dx$
13. $\int x(2x + 3) dx$
14. $\int (2x - 5)(3x + 1) dx$
15. $\int \frac{8x - 5}{\sqrt[3]{x}} dx$
16. $\int \frac{2x^2 - x + 3}{\sqrt{x}} dx$
17. $\int \frac{x^3 - 1}{x - 1} dx$
18. $\int \frac{x^3 + 3x^2 - 9x - 2}{x - 2} dx$
19. $\int \frac{(t^2 + 3)^2}{t^6} dt$
20. $\int \frac{(\sqrt{t} + 2)^2}{t^3} dt$
21. $\int \frac{3}{4} \cos u du$
22. $\int -\frac{1}{5} \sin u du$
23. $\int \frac{7}{\csc x} dx$
24. $\int \frac{1}{4 \sec x} dx$
25. $\int (\sqrt{t} + \cos t) dt$
26. $\int (\sqrt[3]{t^2} - \sin t) dt$
27. $\int \frac{\sec t}{\cos t} dt$
28. $\int \frac{1}{\sin^2 t} dt$
29. $\int (\csc v \cot v \sec v) dv$
30. $\int (4 + 4 \tan^2 v) dv$
31. $\int \frac{\sec w \sin w}{\cos w} dw$
32. $\int \frac{\csc w \cos w}{\sin w} dw$
33. $\int \frac{(1 + \cot^2 z) \cot z}{\csc z} dz$
34. $\int \frac{\tan z}{\cos z} dz$
35. $\int \frac{d}{dx} \sqrt{x^2 + 4} dx$
36. $\int \frac{d}{dx} \sqrt[3]{x^3 - 8} dx$
37. $\int \frac{d}{dx} \sin \sqrt[3]{x} dx$
38. $\int \frac{d}{dx} \sqrt{\tan x} dx$
39. $\frac{d}{dx} \int x^3 \sqrt{x - 4} dx$
40. $\frac{d}{dx} \int x^4 \sqrt[3]{x^2 + 9} dx$
41. $\frac{d}{dx} \int \cot x^3 dx$
42. $\frac{d}{dx} \int \cos \sqrt{x^2 + 1} dx$

Solve the differential equation subject to the given conditions:

43. $f'(x) = 12x^2 - 6x + 1$ $f(1) = 5$

44. $f'(x) = 9x^2 + x - 8$ $f(-1) = 1$

45. $\frac{dy}{dx} = 4x^{1/2}$ $y = 21$ when $x = 4$

Evaluate the following definite integrals:

46. $\int_0^1 2x \, dx$

47. $\int_2^7 3 \, dv$

48. $\int_{-1}^0 (x - 2) \, dx$

49. $\int_2^5 (-3v + 4) \, dv$

50. $\int_{-1}^1 (t^2 - 2) \, dt$

51. $\int_0^3 (3x^2 + x - 2) \, dx$

52. $\int_0^1 (2t - 1)^2 \, dt$

53. $\int_{-1}^1 (t^3 - 9t) \, dt$

54. $\int_1^2 \left(\frac{3}{x^2} - 1 \right) \, dx$

55. $\int_{-2}^{-1} \left(u - \frac{1}{u^2} \right) \, du$

56. $\int_1^4 \frac{u - 2}{\sqrt{u}} \, du$

57. $\int_{-3}^3 v^{1/3} \, dv$

58. $\int_{-1}^1 (\sqrt[3]{t} - 2) \, dt$

59. $\int_1^8 \sqrt{\frac{2}{x}} \, dx$

60. $\int_0^1 \frac{x - \sqrt{x}}{3} \, dx$

61. $\int_0^2 (2 - t)\sqrt{t} \, dt$

62. $\int_{-1}^0 (t^{1/3} - t^{2/3}) \, dt$

63. $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}} \, dx$

64. $\int_0^3 |2x - 3| \, dx$

65. $\int_0^4 |x^2 - 4x + 3| \, dx$

66. $\int_0^\pi (1 + \sin x) \, dx$

67. $\int_0^{\pi/4} \frac{1 - \sin^2 \theta}{\cos^2 \theta} \, d\theta$

68. $\int_{-\pi/6}^{\pi/6} \sec^2 x \, dx$

69. $\int_{\pi/4}^{\pi/2} (2 - \csc^2 x) \, dx$

70. $\int_{-\pi/3}^{\pi/3} 4 \sec \theta \tan \theta \, d\theta$

71. $\int_{-\pi/2}^{\pi/2} (2t + \cos t) \, dt$

72. $\int_1^e \left(2x + \frac{1}{x} \right) \, dx$

73. $\int_1^5 \frac{x + 1}{x} \, dx$

74. $\int_0^2 (e^x + 6) \, dx$

75. $\int_0^3 (t - e^t) \, dt$

76. $\int_{-1}^1 (e^\theta + \sin \theta) \, d\theta$

77. $\int_e^{2e} \left(\cos x - \frac{1}{x} \right) \, dx$

ANSWERS

Indefinite integrals:

- | | | |
|---|--|---------------------------------------|
| 1. $2x^2 + 3x + C$ | 14. $2x^3 - \frac{13x^2}{2} - 5x + C$ | 26. $\frac{3t^{5/3}}{5} + \cos t + C$ |
| 2. $\frac{4x^3}{3} - 4x^2 + x + C$ | 15. $\frac{24x^{5/3}}{5} - \frac{15x^{2/3}}{2} + C$ | 27. $\tan t + C$ |
| 3. $3t^3 - 2t^2 + 3t + C$ | 16. $\frac{4x^{5/2}}{5} - \frac{2x^{3/2}}{3} + 6x^{1/2} + C$ | 28. $-\cot t + C$ |
| 4. $\frac{t^4}{2} - \frac{t^3}{3} + \frac{3t^2}{2} - 7t + C$ | 17. $\frac{x^3}{3} + \frac{x^2}{2} + x + C$ | 29. $-\cot v + C$ |
| 5. $-\frac{z^{-2}}{2} + 3z^{-1} + C$ | 18. $\frac{x^3}{3} + \frac{5x^2}{2} + x + C$ | 30. $4 \tan v + C$ |
| 6. $-\frac{4z^{-6}}{6} + \frac{7z^{-3}}{3} + \frac{z^2}{2} + C$ | 19. $-t^{-1} - 2t^{-3} - \frac{9t^{-5}}{5} + C$ | 31. $\sec w + C$ |
| 7. $2u^{3/2} + 2u^{1/2} + C$ | 20. $-t^{-1} - \frac{8t^{-3/2}}{3} - 2t^{-2} + C$ | 32. $-\csc w + C$ |
| 8. $\frac{2u^{5/2}}{5} + \frac{u^{-1}}{2} + 5u + C$ | 21. $\frac{3}{4} \sin u + C$ | 33. $-\csc z + C$ |
| 9. $\frac{8v^{9/4}}{9} + \frac{24v^{5/4}}{5} - v^{-3} + C$ | 22. $\frac{1}{5} \cos u + C$ | 34. $\sec z + C$ |
| 10. $\frac{v^6}{2} - \frac{3v^{8/3}}{8} + C$ | 23. $-7 \cos x + C$ | 35. $\sqrt{x^2 + 4} + C$ |
| 11. $3x^3 - 3x^2 + x + C$ | 24. $\frac{1}{4} \sin x + C$ | 36. $\sqrt[3]{x^3 - 8} + C$ |
| 12. $\frac{x^3}{3} - 2x - x^{-1} + C$ | 25. $\frac{2t^{3/2}}{3} + \sin t + C$ | 37. $\sin \sqrt[3]{x} + C$ |
| 13. $\frac{2x^3}{3} + \frac{3x^2}{2} + C$ | | 38. $\sqrt{\tan x} + C$ |

Differential equations:

43. $f(x) = 4x^3 - 3x^2 + x + 3$
44. $f(x) = 3x^3 + \frac{x^2}{2} - 8x - \frac{9}{2}$
45. $y(x) = \frac{8x^{3/2}}{3} - \frac{1}{3}$

Definite integrals:

- | | | | |
|---------------------|-----------------------------|--------------------------|-----------------------------------|
| 46. 1 | 54. $\frac{1}{2}$ | 62. $\frac{-27}{20}$ | 70. 0 |
| 47. 15 | 55. -2 | 63. $\frac{1523}{20}$ | 71. 2 |
| 48. $-\frac{5}{2}$ | 56. $\frac{2}{3}$ | 64. $\frac{9}{2}$ | 72. e^2 |
| 49. $-\frac{39}{2}$ | 57. 0 | 65. 4 | 73. $4 + \ln 5$ |
| 50. $-\frac{10}{3}$ | 58. -4 | 66. $\pi + 2$ | 74. $e^2 + 11$ |
| 51. $\frac{51}{2}$ | 59. $2(4 - \sqrt{2})$ | 67. $\frac{\pi}{4}$ | 75. $\frac{11}{2} - e^3$ |
| 52. $\frac{1}{3}$ | 60. $-\frac{1}{18}$ | 68. $\frac{2}{\sqrt{3}}$ | 76. $e - e^{-1}$ |
| 53. 0 | 61. $\frac{16\sqrt{2}}{15}$ | 69. $\frac{\pi}{2} - 1$ | 77. $\sin(2e) - \sin(e) - \ln(2)$ |